

# Algebraic Lie Theory and Representation Theory 2024 (ALTReT2024)

Date: May 17 – 20, 2024

Venue: Kyukamura Minami-Aso (Kumamoto, Japan)

## Abstract

### Special speakers

May 18, 19

- 14:00 – 15:00 Toshiyuki Tanisaki

#### **Quantized flag manifolds and representations of quantum groups**

Abstract: I will give an account of my result on Lusztig's conjectural multiplicity formula for the modules over the De Concini-Kac type quantized enveloping algebras at roots of unity. In the first half I will explain the standard facts on the representation theory of the De Concini-Kac type quantized enveloping algebras at roots of unity. In the second half I will explain how we can use the  $D$ -modules on the quantized flag manifolds in the representation theory of the quantized enveloping algebras.

- 15:30 – 16:30 Masahiko Yoshinaga (Osaka University)

#### **The characteristic polynomial of hyperplane arrangements and related topics**

Abstract: The characteristic polynomial is one of the most important invariants of a hyperplane arrangement. In the first part, I will introduce the notion of characteristic polynomial and survey its connection with combinatorial, topological and algebraic aspects of hyperplane arrangements (minimal CW structure, Aomoto complex, logarithmic vector fields etc). If time permits, we will also discuss June Huh's results on log-concavity. In the second part, we will discuss recent works which are expected to be explored from representation theoretic viewpoints. Especially, we will focus on integral expressions of logarithmic vector fields and its discrete analogue (joint works with Misha Feigin, Daisuke Suyama, Zixuan Wang) and  $q$ -deformation of the Aomoto complex.

## Speakers

May 17

- 15:40 – 16:30 Atsushi Katsuda

#### **Chebotarev density theorem, Quasi-morphism, Modified Riemann-Hilbert problem and Oper**

Abstract: The Chebotarev density theorem gives a density of prime ideals (or its Frobenius class) in a given conjugacy class in the Galois group of algebraic extension of number fields. This is a generalization of the Dirichlet's theorem for counting prime numbers in arithmetic progressions.

Here we consider its geometric analogue for the following three cases: (a)infinite abelian extensions, (b)infinite nilpotent extensions and (c)quasi-morphism version due to P. Sarnak, which is inspired by E. Ghy's lecture at ICM 2006 concerning closed geodesics, their linking number with the trefoil knot, the Rademacher symbol, the Dedekind eta function.

In this talk, we shall explain first the basic strategy of the proof for (a), especially, the Floquet-Bloch theory and

the perturbation of eigenvalues of twisted Laplacians related to the Hodge-de Rham theorem. Next we shall notice several difficulties to generalize to (b) and explain our method to overcome using comparison between unitary representations of discrete nilpotent groups and their Malcev completions, i.e. simply connected nilpotent Lie groups having the previous discrete group as a lattice.

Finally we discuss (c). Modified Riemann Hilbert problem as a substitute of the Hodge-de Rham theorem, Oper(flat vector bundle with possibly non-flat filtrations), and some relations to (b) arise naturally in this case.

- 16:00 – 17:00 Shogo Sugimoto (Waseda University)

**Combinatorics of Demazure characters in type A**

Abstract: Demazure modules are finite-dimensional representations of the Lie ring  $\mathfrak{b}$  of the Borel subgroup  $B$  of a semi-simple Lie group  $G$ , defined using dominant weights and elements of the Weyl group. Key polynomials are special polynomials representing the characters of Demazure modules in type A, defined inductively using operators called Demazure operators. Assaf provided an explicit formula for the Key polynomials using tableaux called semi-standard key tableaux. In this talk, we will discuss Assaf’s provided tableau formula and an alternative proof of this formula. This talk is based on a joint work with Tomoo Matsumura.

- 20:00 – 20:50 Yusuke Nakayama (Waseda University)

**Homogeneous Ulrich bundles on homogenous varieties of certain exceptional type**

Abstract: There is an intriguing problem to investigate whether every smooth projective variety carries an Ulrich bundle with respect to some polarization, as posed by Eisenbud, Schreyer, and Weyman. We provide a criterion for an initialized irreducible homogeneous vector bundle on a rational homogeneous variety with any Picard number to be Ulrich with respect to any polarizations. As an application, we show that rational homogeneous varieties with Picard number at least two of certain exceptional algebraic groups do not admit such homogeneous Ulrich bundles with respect to the minimal ample class.

**May 18**

- 9:10 – 10:00 Duc-Khanh Nguyen (OIST)

**On the shifted Littlewood-Richardson coefficients and Littlewood-Richardson coefficients**

Abstract: We give a new interpretation of the shifted Littlewood-Richardson coefficients  $f_{\lambda\mu}^\nu$  ( $\lambda, \mu, \nu$  are strict partitions). The coefficients  $g_{\lambda\mu}$  which appear in the decomposition of Schur  $Q$ -function  $Q_\lambda$  into the sum of Schur functions  $Q_\lambda = 2^{l(\lambda)} \sum_{\mu} g_{\lambda\mu} s_\mu$  can be considered as a special case of  $f_{\lambda\mu}^\nu$  (here  $\lambda$  is a strict partition of length  $l(\lambda)$ ).

We also give another description for  $g_{\lambda\mu}$  as the cardinal of a subset of a set that counts Littlewood-Richardson coefficients  $c_{\mu^t\mu}^{\tilde{\lambda}}$ . This new point of view allows us to establish connections between  $g_{\lambda\mu}$  and  $c_{\mu^t\mu}^{\tilde{\lambda}}$ . More precisely, we prove that  $g_{\lambda\mu} = g_{\lambda\mu^t}$ , and  $g_{\lambda\mu} \leq c_{\mu^t\mu}^{\tilde{\lambda}}$ . We conjecture that  $g_{\lambda\mu}^2 \leq c_{\mu^t\mu}^{\tilde{\lambda}}$  and formulate some conjectures on our combinatorial models which would imply this inequality if it is valid.

- 10:15 – 11:05 Atsuro Yoshida (Nagoya University)

**Orbit structures of some posets arising from Lie theory under rowmotion**

Abstract: Dynamical algebraic combinatorics is a research area in combinatorics where they study phenomena observed when an action is repeatedly applied to a set of combinatorial objects such as tableaux, order ideals of finite posets, words, etc. One of the most well-studied actions in this field is rowmotion. To date, various kinds of rowmotion have been studied, and among them, those on the combinatorial, plane partitions, piecewise linear, and birational levels have been getting great attention. In their paper, Hopkins proposed a series of conjectures about rowmotion, suggesting the idea that the posets of a minuscule doppelgänger pair behave under rowmotion as if they had isomorphic comparability graphs. In this talk, we give a decomposition method that works for ordinal sums of antichains and reduces the action of plane partition rowmotion to that of combinatorial rowmotion, and then we explain its application to one of the conjectures by Hopkins.

- 11:20 – 12:10 Masato Tanaka (Nagoya University)  
**Solvability and nilpotency of Kac–Paljutkin’s finite quantum group and Sekine quantum groups**  
 Abstract: TBA
- 16:50 – 17:40 Yoritaka Iwata (Osaka University of Economics and Law)  
**Algebraic module of unbounded infinitesimal generators**  
 Abstract: [https://nfujita-math.github.io/altret2024/iwata\\_ALReT\\_abstract.pdf](https://nfujita-math.github.io/altret2024/iwata_ALReT_abstract.pdf)
- 20:00 – 20:30 Naoya Hiramae (Kyoto University)  
**On  $\tau$ -tilting finiteness of group algebras**  
 Abstract: Demonet–Iyama–Jasso introduced a new class of finite dimensional algebras,  $\tau$ -tilting finite algebras, and showed that  $\tau$ -tilting finiteness of algebras relates to brick finiteness and functorially finiteness of torsion classes. To date, many researchers have studied  $\tau$ -tilting finite algebras. In the context of modular representation theory of finite groups, Eisele–Janssens–Raedschelders showed that group algebras of tame type are  $\tau$ -tilting finite. Given the classical result that the representation type of group algebras is determined by their  $p$ -Sylow subgroups, where  $p$  denotes the characteristic of the ground field, it is natural to ask what controls  $\tau$ -tilting finiteness of group algebras. In this talk, we will see that  $\tau$ -tilting finiteness of group algebras is determined by so-called  $p$ -hyperfocal subgroups in some special cases. This talk is based on a joint work with Yuta Kozakai.
- 20:35 – 21:05 Ana Kontrec (RIMS)  
**Kazama–Suzuki duality between certain simple W-algebras**  
 Abstract: TBA

## May 19

- 9:10 – 10:00 Taito Tauchi (Aoyama Gakuin University)  
**Double flag variety for indefinite unitary group  $U(p, p)$**   
 Abstract: Let  $G$  be a reductive algebraic group and  $H$  its symmetric subgroup. Then, a projective variety having the form  $X = H/P_H \times G/P_G$  is called a double flag variety, where  $P_H$  and  $P_G$  are parabolic subgroups of  $H$  and  $G$ , respectively. Although there are many studies of the theory of double flag varieties over the complex number field, there does not seem to be much research in the case over the real number field. In this talk, we discuss the orbit structure of a real double flag variety in the case  $(G, H, P_G, P_H) = (U(p, p), GL_p(\mathbb{C}), P_S, B_H)$ , where  $P_S$  is the Siegel parabolic subgroup of  $G$ , and  $B_H$  is a Borel subgroup of  $H$ . An interesting point in this case is that  $H \backslash X$  can be regarded as a family of the orbit spaces  $U(m, n) \backslash GL_k(\mathbb{C}) / B_k$  for  $1 \leq k = m + n \leq p$ , where  $B_k$  is a Borel subgroup of  $GL_k(\mathbb{C})$ . This talk is based on a joint work with Kyo Nishiyama.
- 10:15 – 11:05 Shunsuke Kano (Tohoku University)  
**Entropy of cluster DT transformations and the finite-tame-wild trichotomy for acyclic quivers**  
 Abstract: The cluster algebra associated with an acyclic quiver has a special mutation loop  $\tau$ , called the cluster Donaldson–Thomas (DT) transformation, related to the Auslander–Reiten translation. In this talk, I will characterize the finite-tame-wild trichotomy for acyclic quivers by the sign stability of  $\tau$ , introduced by the speaker and Tsukasa Ishibashi, and its cluster stretch factor. As an application, we will compute several kinds of entropies of  $\tau$  and other mutation loops. In particular, we will show that any mutation loop of finite or tame acyclic quivers has zero algebraic entropy.
- 11:20 – 12:10 Tsukasa Ishibashi (Tohoku University)  
**Local acyclicity of the cluster algebras from moduli spaces of G-local systems**  
 Abstract: We show that the cluster algebras arising from the moduli spaces of G-local systems are locally acyclic, refining the previous A=U result. A key geometric ingredient is a construction of a covering of the moduli space by open subspaces related to the braid variety. The talk is based on a joint work in progress with Hironori Oya

and Linhui Shen.

- 16:50 – 17:40 Hiroyuki Yamane (University of Toyama)

**Hamiltonian cycles and generalized quantum groups**

Abstract: My talk will be composed of the three parts:

(1) I briefly explain notions of completions of  $h$ -adic and metric topologies.

(2) I explain the typical irreducible character formula for a generalized quantum group  $U$  (J. Algebra Appl. 20 (2021), no. 1, 2140014). There are two finite dimensional irreducible highest weight  $U$  modules  $V(\lambda)$  and  $V(\mu)$  such that  $V(\lambda)$  is typical and  $V(\mu)$  is not typical, their highest weights  $\lambda$  and  $\mu$  looks very similar, but their characters are different (Appendix of arXiv:1909.08881).

(3) I also explain Hamiltonian cycles of the Cayley graphs of the finite Weyl groupoids (arXiv:2310.12543 (see also Toyama. Math. J. 43 (2022), 1-76)). This part is a joint work with Takato Inoue.

## May 20

- 9:00 – 9:50 Takashi Kouno (Waseda University)

**Positivity conjecture for the equivariant quantum  $K$ -ring of some partial flag manifolds**

Abstract: We show the positivity property of Schubert structure constants in the Chevalley formulas for the equivariant quantum  $K$ -rings of some partial flag manifolds. In particular, we consider the full flag manifold in arbitrary types, the two-step flag manifold in type  $A$ , and the isotropic Grassmannian (type  $C$ ). Based on the explicit Chevalley formula described in the recent works of Lenart–Naito–Sagaki and Kouno–Lenart–Naito–Sagaki, we can write down the structure constants combinatorially. Thanks to this description, we can derive the desired positivity property. This talk is based on the joint work with Cristian Lenart, Satoshi Naito, and Daisuke Sagaki.

- 10:35 – 11:25 Takeshi Ikeda (Waseda University)

**Equivariant  $K$ -homology of affine Grassmannian**

Abstract: Let be  $G$  a simply connected simple complex linear algebraic group. We are interested in the torus equivariant  $K$ -homology ring  $K_*^T(\mathrm{Gr}_G)$  of the affine Grassmannian of  $G$ , which is closely related to the equivariant quantum  $K$ -ring  $QK_T(G/B)$  of the flag variety  $G/B$ . For  $G = SL_n$ , we provide two realizations of  $K_*^T(\mathrm{Gr}_G)$ ; one is a coordinate ring of a centralizer family  $\mathcal{Z}$  for  $PGL_n$ , and the other is a certain deformed ring of symmetric functions. We also show how an integrable system called the relativistic Toda lattice gives rise to an isomorphism between the coordinate ring of  $\mathcal{Z}$  and  $QK_T(SL_n/B)$  up to some localizations. The goal of this talk is to give a correspondence between the Schubert bases of the both rings in terms of explicit combinatorial counterparts. This talk is based on joint work with Shinsuke Iwao, Satoshi Naito, Mark Shimozono, and Kohei Yamaguchi.

Organizers:

Naoki Fujita (Kumamoto University)

Katsuyuki Naoi (Tokyo University of Agriculture and Technology)